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# Annihilation Production $p+\bar{p} \rightarrow \Lambda+\bar{\Lambda}$

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The reaction $p+\bar{p} \rightarrow \Lambda+\bar{\Lambda}$ is considered in this paper in full detail with the assumption that the process is dominated by the exchange of a $K^*$. Both the cases of an elementary and of a Regge-pole $K^*$ are discussed. Results on differential cross section, polarization functions, and angular correlation functions with various combinations of longitudinal and transverse polarizations of $\Lambda$ and $\bar{\Lambda}$ are presented. These values can be checked easily in a bubble chamber experiment. We find that from the differential cross section alone it is impossible to distinguish one type of $K^*$ from the other, but a measurement of the angular correlations may give deciding information.

## I. INTRODUCTION

It has been shown recently, by both the CERN[1] and Brookhaven[2] groups, in the hydrogen bubble chamber experiment for the annihilations $p+\bar{p} \rightarrow \Lambda+\bar{\Lambda}$, $p+\bar{p} \rightarrow \Lambda+\bar{\Sigma}^0$, and $p+\bar{p} \rightarrow \Sigma^0+\bar{\Lambda}$ in the energy range 3–4 BeV/$c$, that the antihyperon is produced predominantly in the forward direction. Bessis, Itzykson, and Jacob,[3] (BIJ) and several others[4] independently, using a formalism based on some peripheral model, obtained a good fit with the experimental results. They suggested that the annihilation process $p+\bar{p} \rightarrow Y+\bar{Y}$ is dominated by the exchange of an isospin-$\frac{1}{2}$ and strangeness-1 boson between the baryon pairs (Fig. 1). A particle with these quantum numbers can either be a $K$ or a $K^*$, but only an exchange of a vector $K^*$ gives the right angular distribution. BIJ[3] have also proposed a simple test of this mechanism based on an angular correlation measurement between the directions of the pions emitted in the hyperon and antihyperon decays. This is subject to a simple experimental measurement and is, in fact, being carried out by the Ecole Polytechnique group.

Though the peripheral model gives a right fit to the experimental results, it has the defect of violating unitarity at high energy because of vector boson exchange. Regge formalism, on the other hand, avoids such a difficulty and it also meets with some success in explaining the $N$–$N$ and $N$–$\bar{N}$ scatterings at high energies.[5] In this paper, we therefore attempt to re-examine our problem in more detail by considering both cases—(i) exchange of an elementary $K^*$ and (ii) exchange of a Regge-pole $K^*$—in the hope of knowing whether the present experimental data are in favor of or against the second type of exchange.

In our paper we shall consider $p+\bar{p} \rightarrow \Lambda+\bar{\Lambda}$ only. In Sec. II we give the general formalism and the kinematics—the latter being complicated as is always the case when spins are involved. However, once we have worked out the kinematics, it is then easy to write the differential cross section, polarization functions, and angular correlation functions. The polarizations of the lambda and antilambda in our problem are easily observed in a bubble chamber experiment. In Sec. III the model of the exchange of a single elementary $K^*$ is reconsidered, differential cross section and angular correlation functions are calculated, and in Figs. 4 and 5 curves are plotted, at two different energies. In Sec. IV we examine the case when a Regge-pole $K^*$ is exchanged. All spins are taken into account but no attempt is made to work out the analyticity properties of the $S$ matrix in the complex angular momentum plane, and we shall assume that the $S$ matrix is meromorphic in the right half-plane, at least for $\mathrm{Re}\,j \geqslant -\frac{1}{2}$.

In the last section the results are discussed. We conclude that the present experimental situation cannot distinguish between the two models. However, we see that measurements of angular correlations and total cross sections at higher energies will give decisive information.

## II. KINEMATICS

We restrict ourselves to the annihilation of $p+\bar{p} \rightarrow \Lambda+\bar{\Lambda}$. Other hyperon-antihyperon pair productions will not be considered as they have similar properties. For this process, the $T$ matrix can be expressed in the form

$$\begin{aligned}T=&F_1\bar{v}(p_2)u(p_1)\bar{u}(-q_1)v(-q_2)\\&+F_2\bar{v}(p_2)\gamma_\mu u(p_1)\bar{u}(-q_1)\gamma_\mu v(-q_2)\\&+\tfrac{1}{2}F_3\bar{v}(p_2)\sigma_{\mu\nu}u(p_1)\bar{u}(-q_1)\sigma_{\mu\nu}v(-q_2)\\&+F_4\bar{v}(p_2)i\gamma_5\gamma_\mu u(p_1)\bar{u}(-q_1)i\gamma_5\gamma_\mu v(-q_2)\\&+F_5\bar{v}(p_2)\gamma_5 u(p_1)\bar{u}(-q_1)\gamma_5 v(-q_2)\\&+F_6\tfrac{1}{2}(q_2-q_1)_\mu\bar{v}(p_2)i\gamma_\mu u(p_1)\bar{u}(-q_1)v(-q_2)\,,\end{aligned}\qquad(1)$$

where $p_1$, $p_2$ and $-q_1$, $-q_2$ are the 4-momenta of the incoming proton, antiproton and outgoing lambda and antilambda, respectively (Fig. 2). The $F$'s ($s$ form 1 to

[1] R. Armenteros *et al.*, *Proceedings of the 1962 International Conference on High-Energy Physics at CERN* (CERN, Geneva, 1962), p. 236.

[2] C. Baltay *et al.*, *Proceedings of the 1962 International Conference on High-Energy Physics at CERN* (CERN, Geneva, 1962), p. 233.

[3] D. Bessis, C. Itzykson, and M. Jacob, Nuovo Cimento **27**, 376 (1963).

[4] H. D. D. Watson, Imperial College preprint, ICTP-63-15 (unpublished).

[5] G. Cocconi and S. D. Drell, *Proceedings of the 1962 International Conference on High-Energy Physics at CERN* (CERN, Geneva, 1962), pp. 883 and 897.

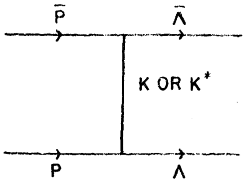


FIG. 1. One-particle exchange diagram for the annihilation process $p+\bar{p}\rightarrow\Lambda+\bar{\Lambda}$.

6) are invariant functions of $s$, $t$, and $u$ where

$$s=-(p_1+p_2)^2,$$
$$t=-(p_1+q_1)^2,$$
$$u=-(p_1+q_2)^2,$$
$$s+t+u=2M^2+2\Lambda^2.$$

In the center-of-mass system

$$s=4E^2=W^2,$$
$$t=M^2+\Lambda^2-2E^2+2pq\cos\theta,$$
$$u=M^2+\Lambda^2-2E^2-2pq\cos\theta,$$

where $E$ is the energy, and $p$ and $q$ are the momenta of the proton and the lambda; $\theta$ is the scattering angle defined as

$$x=\cos\theta=\mathbf{p}_1\cdot\mathbf{q}_1/pq,$$

i.e., the angle between the incoming proton and the outgoing lambda.

For our purpose it is useful to diagonalize the $T$ matrix in helicity states.[6] We define

$$\phi_i=(M\Lambda/4\pi E)\langle\lambda_\Lambda,\lambda_{\bar{\Lambda}}|T|\lambda_p,\lambda_{\bar{p}}\rangle. \qquad (2)$$

Because of parity and charge conjugation invariances, there are only six independent helicity amplitudes (the number of these does agree with the number of scalar invariants) which we define as shown in Table I. The following relations between $\phi$'s and $F$'s can be obtained by using standard methods involving, however, tedious algebra (see Appendix):

$$\begin{aligned}
4\pi E\phi_1&=pqF_1-M\Lambda xF_2-(E^2-pq)xF_3\\
&\quad-M\Lambda F_4-E^2F_5-Mq^2xF_6,\\
4\pi E\phi_2&=pqF_1-M\Lambda xF_2-(E^2+pq)xF_3\\
&\quad+M\Lambda F_4+E^2F_5-Mq^2xF_6,\\
4\pi E\phi_3&=-E^2(1+x)F_2-M\Lambda(1+x)F_3\\
&\quad+pq(1+x)F_4,\\
4\pi E\phi_4&=-E^2(1-x)F_2-M\Lambda(1-x)F_3\\
&\quad-pq(1-x)F_4,\\
4\pi E\phi_5&=\Lambda E\sin\theta F_2+ME\sin\theta F_3+Eq^2\sin\theta F_6,\\
4\pi E\phi_6&=-ME\sin\theta F_2-\Lambda E\sin\theta F_3.
\end{aligned} \qquad (3)$$

We note that the matrix in (3) is nonsingular. 'his guarantees that our choice of $\phi$'s and $F$'s is correct.

Our purpose of introducing helicity amplitudes is that with the help of them we can immediately write down the differential cross section, polarization functions, and angular correlation functions. These functions can be easily measured in a bubble chamber by measuring the decay angles of the emitted $\pi^-$ (and $\pi^+$) of $\Lambda$ (and $\bar{\Lambda}$). We shall, for the purpose of analyzing, introduce a set of three unit right-handed vectors $(n_1,n_2,n_3)$, where $n_3$ is along the direction of $\mathbf{q}_1$, and $n_2$ in the direction of $\mathbf{p}_1\times\mathbf{q}_1$ (Fig. 3). The angular correlation function $R_{n_i\bar{n}_j}(\theta)$ at a fixed scattering angle $\theta$ is taken as the probability of finding $\Lambda$ polarized in the $n_i$ direction and simultaneously $\bar{\Lambda}$ polarized in the $-n_j$ direction, the polarization functions $Q_{n_i}(\theta)$ as the probability of finding $\Lambda$ polarized along $n_i$ when the polarization of $\bar{\Lambda}$ is unobserved, similarly $Q_{\bar{n}_j}(\theta)$ as the probability of finding $\bar{\Lambda}$ polarized along $-n_j$ and the polarization of $\Lambda$ is unobserved.

We consider the case when the beam and the target are unpolarized as is generally true in these experiments. Under this assumption most of the nine angular correlation functions[7] and the six polarization functions are zero. The remaining ones can be easily calculated[8] from (2) and Table I (see the Appendix). The results are as follows:

$$\begin{aligned}
&d\sigma/d\Omega=\tfrac{1}{2}(q/p)X,\\
&Q_{n_1}(\theta)=Q_{n_3}(\theta)=Q_{\bar{n}_1}(\theta)=Q_{\bar{n}_3}(\theta)=0,\\
&Q_{n_2}(\theta)=-Q_{\bar{n}_2}(\theta)=(2/X)\operatorname{Im}[(\phi_1+\phi_2)\phi_6^*\\
&\qquad-(\phi_3-\phi_4)\phi_5^*],\\
&R_{n_1\bar{n}_1}(\theta)=(2/X)\operatorname{Re}[\phi_6\phi_6^*-\phi_3\phi_4^*-\phi_1\phi_2^*-\phi_5\phi_5^*],\\
&R_{n_2\bar{n}_2}(\theta)=(2/X)\operatorname{Re}[-\phi_6\phi_6^*+\phi_3\phi_4^*\\
&\qquad-\phi_1\phi_2^*-\phi_5\phi_5^*],\\
&R_{n_3\bar{n}_3}(\theta)=(1/X)\operatorname{Re}[|\phi_1|^2+|\phi_2|^2+2|\phi_5|^2\\
&\qquad-|\phi_3|^2-|\phi_4|^2-2|\phi_6|^2],\\
&R_{n_1\bar{n}_3}(\theta)=R_{n_3\bar{n}_1}(\theta)=(-2/X)\operatorname{Re}[(\phi_1+\phi_2)\phi_6^*\\
&\qquad+(\phi_3-\phi_4)\phi_5^*],\\
&R_{n_1\bar{n}_2}(\theta)=R_{n_2\bar{n}_1}(\theta)=R_{n_2\bar{n}_3}(\theta)=R_{n_3\bar{n}_2}(\theta)=0,
\end{aligned} \qquad (4)$$

where

$$X=[|\phi_1|^2+|\phi_2|^2+|\phi_3|^2+|\phi_4|^2+2|\phi_5|^2+2|\phi_6|^2].$$

All these functions can be experimentally measured. For instance, $-\frac{1}{4}\alpha^2R_{n_i\bar{n}_j}(\theta)$, for each fixed $\theta$, is the ratio of the difference of the number of times a $\pi^-$ is emitted

TABLE I. Helicity amplitudes for the $p+\bar{p}\rightarrow\Lambda+\bar{\Lambda}$ annihilation.

| $\Lambda\bar{\Lambda}$ \ $p\bar{p}$ | $++$ | $+-$ | $-+$ | $--$ |
|---|---|---|---|---|
| $++$ | $\phi_1$ | $\phi_5$ | $-\phi_5$ | $\phi_2$ |
| $+-$ | $\phi_6$ | $\phi_3$ | $\phi_4$ | $\phi_6$ |
| $-+$ | $-\phi_6$ | $\phi_4$ | $\phi_3$ | $-\phi_6$ |
| $--$ | $\phi_2$ | $\phi_5$ | $-\phi_5$ | $\phi_1$ |

[6] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) 7, 404 (1959).

[7] The author wishes to thank Professor Michel for pointing out this fact.

[8] The function $R_{n_3\bar{n}_3}(\theta)$ has also been given by BIJ.

above and below the plane orthogonal to $n_i$ to the total $\pi^-$ emissions when a $\pi^+$ is observed above the plane normal to $-n_j$. Again the functions $\frac{1}{2}\alpha Q_{n_i}(\theta)$ $[-\frac{1}{2}\alpha Q_{\bar{n}_j}(\theta)]$ are the ratios of the difference of the number of times a $\pi^-(\pi^+)$ is emitted above and below the plane orthogonal to $n_i(-n_j)$ to the total $\pi^-(\pi^+)$ emissions. The parameter $\alpha$ is the asymmetry parameter for $\Lambda\rightarrow p+\pi^-$ decay and has been previously measured to be $-0.62\pm0.07$.[9] We also note here that because of the $CP$ invariance, the asymmetry parameter for the $\bar{\Lambda}$ decay is opposite to that of the $\Lambda$ decay.

From (4) we see four of the angular correlation functions are zero, this is because we have used the assumption that the beam and target are unpolarized. The vanishing of these functions can in fact provide a test of the extent to which the beam $\bar{p}$ is unpolarized. On the other hand, if these functions are found to be zero, they do not prove that our beam is unpolarized.

### III. ON THE EXCHANGE OF AN ELEMENTARY $K^*$

It has been shown in the last section that if we could by some means estimate the values of the scalar invariants based on some model, then we can immediately relate them by (3) and (4) to the experimentally readily observable quantities and hence the model can be tested.

In the following we shall investigate the problem under the assumption that the process is dominated by the exchange of a $K^*$. We shall first treat $K^*$ as an elementary particle and then consider the case when it is a Regge pole. We then compare these two results. The diagram for the exchange of a $K$ is believed to be unimportant in our case; both phenomenologically because it does not give a correct angular distribution and theoretically because the $K$ Regge trajectory is much lower than that of $K^*$ and hence it is not important at high energy.

The model involving the exchange of an elementary $K^*$ has been considered by previous authors. However, we would like to reproduce their results here and consider them in a little more detail. The most general expression for the $K^*N\Lambda$ vertex reads as[10]

$$\bar{u}(\Lambda)[G(\Delta^2)\gamma_\mu+G'(\Delta^2)\sigma_{\mu\nu}\Delta_\nu+G''(\Delta^2)\Delta_\mu]u(p),$$

where $\Delta_\mu$ is the 4-momentum transfer. The functions

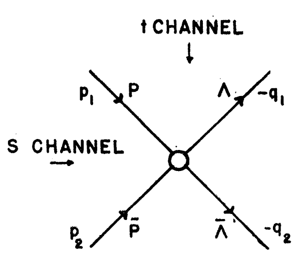


FIG. 2. The production $p+\bar{p}\rightarrow\Lambda+\bar{\Lambda}$ in the $s$ channel or the elastic scattering $p+\bar{\Lambda}\rightarrow p+\bar{\Lambda}$ in the $t$ channel.

[9] J. W. Cronin and O. E. Overseth, Phys. Rev. **129**, 1795 (1963).
[10] We have taken the relative parity of $N\Lambda$ to be even.

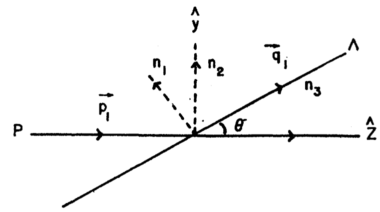


FIG. 3. The production $p+\bar{p}\rightarrow\Lambda+\bar{\Lambda}$ in the c.m. system. $\mathbf{p}_1$ is in the $z$ direction and $\mathbf{q}_1$ is in the $x$-$z$ plane; $n_2$ is normal to the production plane.

$G^i(\Delta^2)$ are real because of time reversal invariance. For a first-order approximation the last two couplings can be neglected for small values of $\Delta^2$ which is the case for forward scattering. We can therefore write down the matrix element for the diagram of Fig. 1 with the exchange of an elementary $K^*$.

$$T=[G(\Delta^2)]^2\bar{u}(-q_1)\gamma_\mu u(p_1)\times\frac{g_{\mu\nu}-(\Delta_\mu\Delta_\nu/m^2)}{-t+m^2}\bar{v}(p_2)\gamma_\nu v(-q_2),$$

where $m$ is the mass of $K^*$. Comparing it with (2), we immediately obtain

$$\begin{aligned}
\phi_1&=\frac{1}{4\pi E}\frac{[G(\Delta^2)]^2}{m^2-t}[2E^2-2M^2+\tfrac{1}{2}M^2(1+x)],\\
\phi_2&=\frac{1}{4\pi E}\frac{[G(\Delta^2)]^2}{m^2-t}[-\tfrac{1}{2}M^2(1-x)],\\
\phi_3&=\frac{1}{4\pi E}\frac{[G(\Delta^2)]^2}{m^2-t}[\tfrac{1}{2}(2E^2-M^2)(1+x)],\\
\phi_4&=-\phi_2,\\
\phi_5&=\frac{1}{4\pi E}\frac{[G(\Delta^2)]^2}{m^2-t}[-\tfrac{1}{2}EM\sin\theta],\\
\phi_6&=-\phi_5.
\end{aligned}\tag{5}$$

In obtaining (5) we have neglected the mass difference between the proton and the lambda, which is indeed negligible at high energy considered in our case. Using (4), various angular correlation functions are computed at the energies 3.3 and 7.5 GeV/$c$, respectively. Curves are plotted in Figs. 4 and 5. Their shapes do resemble each other though they differ in magnitude. The calculation of these functions at the energy 7.5 GeV/$c$, which is much higher than those used in previous experiments, is for the purpose of comparing them later on with those based on a Regge-pole $K^*$.

We have also computed the differential cross section for these two energies. In the calculation we have approximated the form factor $G(\Delta^2)$ as a constant, this approximation being valid for a small range of $t$ near the forward peak. These curves are also presented in Figs. 4 and 5. They are normalized to 1 at $\cos\theta=1$. Both curves are peaked in the forward direction though

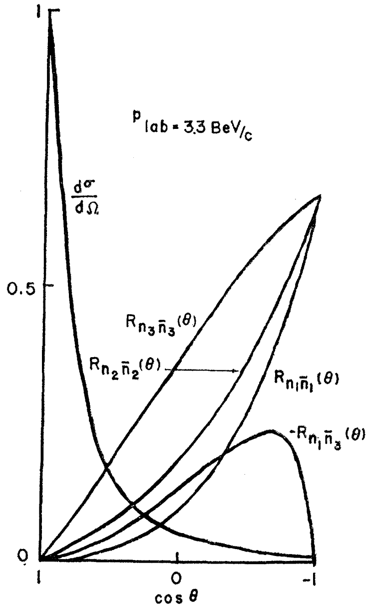


FIG. 4. Curves for the differential cross section (normalized in the forward direction) and various angular correlations for the production $p+\bar{p}\rightarrow\Lambda+\bar{\Lambda}$ at the energy 3.3 BeV/$c$ based on the one-elementary $K^*$-exchange model.

the peak at higher energy is more pronounced than the other.

We would like to emphasize that the curves we have presented are not to be taken seriously in the region of large momentum transfer (or large angle scattering). This is due to two reasons: (i) the exchange of a $K^*$ does not dominate backward scattering, (ii) the form factor $G(\Delta^2)$ can no longer be approximated as a constant for large $t$.

## IV. ON THE EXCHANGE OF A REGGE-POLE $K^*$

We shall now consider a Regge-pole $K^*$. The exchange of a Regge-pole $K^*$ in the $p+\bar{p}\rightarrow\Lambda+\bar{\Lambda}$ annihilation can be pictured as a pole in the crossed channel. Hence, we investigate the elastic scattering $p+\bar{\Lambda}\rightarrow p+\bar{\Lambda}$ and then extend our result to the $s$ channel by analytic continuation. The $T$ matrix for the $p+\bar{\Lambda}\rightarrow p+\bar{\Lambda}$ scattering has a form similar to (1) and can be obtained form (1) simply by replacing $v(p_2)$ by $\bar{u}(-p_2)$ and $\bar{u}(-q_1)$ by $v(q_1)$. The $F$'s being invariant analytic functions of $s$, $t$, $u$, do not change under this replacement.

In the c.m. system of the $t$ channel

$$s=-2k^2(1-\cos\bar{\theta})\,,$$
$$t=\bar{w}^2=(E_p+E_\Lambda)^2\,,$$
$$u=M^2+\Lambda^2-2E_pE_\Lambda-2k^2\cos\bar{\theta}\,,$$

where $k$ is the momentum, $Ep$ and $E_\Lambda$ are the center-of-mass energy of the proton and the antilambda, respectively, $\bar{\theta}$ is the scattering angle and is defined as $z=\cos\theta=\mathbf{p}_1\cdot\mathbf{p}_2/k^2$, i.e., the scattering angle between the incoming and outgoing protons, $\bar{w}$ is the total energy.

We shall again introduce helicity amplitudes because they have the property that they can be projected easily into partial waves and extended to the complex angular-momentum plane by the Sommerfeld-Watson transformation. Because of $P$ and $T$ invariances, the number of independent helicity amplitudes is 6, which we define as shown in Table II.

TABLE II. Helicity amplitudes for the scattering $p+\bar{\Lambda}\rightarrow p+\bar{\Lambda}$.

| $p\bar{\Lambda}_{\text{out}}$ \ $p\bar{\Lambda}_{\text{in}}$ | $++$ | $+-$ | $-+$ | $--$ |
|---|---|---|---|---|
| $++$ | $\chi_1$ | $\chi_5$ | $\chi_6$ | $\chi_2$ |
| $+-$ | $-\chi_5$ | $\chi_3$ | $\chi_4$ | $\chi_6$ |
| $-+$ | $-\chi_6$ | $\chi_4$ | $\chi_3$ | $\chi_5$ |
| $--$ | $\chi_2$ | $-\chi_6$ | $-\chi_5$ | $\chi_1$ |

The functions $\chi_s$ are normalized as follows:

$$\chi_i=2M\Lambda\langle\lambda_p\lambda_{\bar{\Lambda}}|T|\lambda_p\lambda_{\bar{\Lambda}}\rangle\,,\qquad i=1,2,3,4$$

$$\chi_i=2M\Lambda\frac{M+\Lambda}{\bar{w}\sin\bar{\theta}}\langle\lambda_p\lambda_{\bar{\Lambda}}|T|\lambda_p\lambda_{\bar{\Lambda}}\rangle\,,\quad i=5,6\,.$$

The reason for choosing such a normalization is that the $\chi$'s and the $F$'s then have the same analytic property, as can be seen below in (6).

The six helicity amplitudes are related to the six scalar invariant functions—this can be done easily but

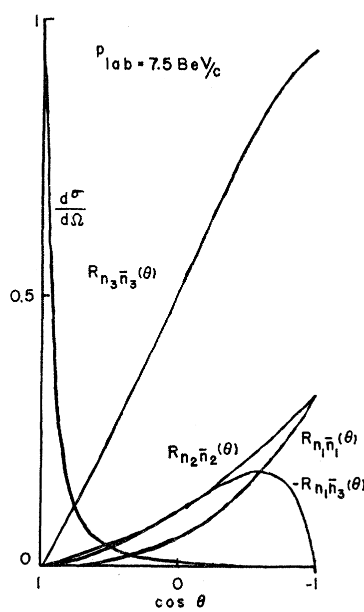


FIG. 5. Same as in Fig. 4, but at the energy of 7.5 BeV/$c$.

the algebra is complex:

$$\begin{aligned}
\chi_1 &= -M\Lambda(1+z)F_1+[E_pE_\Lambda+3k^2+(E_pE_\Lambda-k^2)z]F_2-M\Lambda(3-z)F_3 \\
&\quad +[3E_pE_\Lambda+k^2-(E_pE_\Lambda-k^2)z]F_4-\Lambda(E_pE_\Lambda+k^2)(1+z)F_6\,, \\
\chi_2 &= E_pE_\Lambda(1-z)F_1-M\Lambda(1-z)F_2+(E_pE_\Lambda+k^2)(3+z)F_3 \\
&\quad -M\Lambda(3+z)F_4+k^2(1-z)F_5+ME_\Lambda{}^2(1-z)F_6\,, \\
\chi_3 &= -M\Lambda(1+z)F_1+(E_pE_\Lambda+k^2)(1+z)F_2+M\Lambda(1+z)F_3 \\
&\quad -(E_pE_\Lambda+k^2)(1+z)F_4-\Lambda(E_pE_\Lambda+k^2)(1+z)F_6\,, \\
\chi_4 &= -E_pE_\Lambda(1-z)F_1+M\Lambda(1-z)F_2+(E_pE_\Lambda-k^2)(1-z)F_3 \\
&\quad -M\Lambda(1-z)F_4+k^2(1-z)F_5-ME_\Lambda{}^2(1-z)F_6\,, \\
\bar{w}\chi_5/(M+\Lambda) &= ME_\Lambda F_1-\Lambda E_pF_2-ME_\Lambda F_3+\Lambda E_pF_4+E_\Lambda(E_\Lambda E_p+k^2)F_6\,, \\
\bar{w}\chi_6/(M+\Lambda) &= -\Lambda E_pF_1+ME_\Lambda F_2+\Lambda E_pF_3-ME_\Lambda F_4-M\Lambda E_\Lambda F_6\,.
\end{aligned} \tag{6}$$

This set of linear relations are inverted because we are interested in estimating $F$'s from $\chi$'s.

$$\begin{aligned}
4k^2tF_1 &= M\Lambda\chi_1+(E_pE_\Lambda+k^2)\chi_2+\frac{M}{\Lambda}\left(\frac{4E_pE_\Lambda}{1+z}+E_\Lambda{}^2+k^2\right)\chi_3-(E_pE_\Lambda+k^2)\chi_4-\frac{2M\bar{w}}{M+\Lambda}E_\Lambda(1-z)\chi_5 \\
&\quad -\frac{2\bar{w}}{\Lambda(M+\Lambda)}(E_pE_\Lambda+k^2)[2E_p+E_\Lambda(1+z)]\chi_6\,, \\
4k^2tF_2 &= (E_pE_\Lambda+k^2)\chi_1+M\Lambda\chi_2+\left(\frac{4E_\Lambda{}^2}{1+z}+E_pE_\Lambda-k^2\right)\chi_3-M\Lambda\chi_4 \\
&\quad +\frac{2\Lambda\bar{w}}{M+\Lambda}[2E_\Lambda+E_p(1+z)]\chi_5+\frac{2M\bar{w}}{M+\Lambda}E_\Lambda(1-z)\chi_6\,, \\
4k^2tF_3 &= M\Lambda\chi_1+(E_pE_\Lambda+k^2)\chi_2+\frac{M}{\Lambda}\left(-\frac{4E_\Lambda{}^2}{1+z}+E_\Lambda{}^2+k^2\right)\chi_3-(E_pE_\Lambda+k^2)\chi_4 \\
&\quad -\frac{2M\bar{w}}{M+\Lambda}E_\Lambda(1-z)\chi_5+\frac{2\bar{w}E_\Lambda}{\Lambda(M+\Lambda)}(E_pE_\Lambda+k^2)(1-z)\chi_6\,, \\
4k^2tF_4 &= (E_pE_\Lambda+k^2)\chi_1+M\Lambda\chi_2+\left(-\frac{4E_pE_\Lambda}{1+z}+E_pE_\Lambda-k^2\right)\chi_3-M\Lambda\chi_4 \\
&\quad -\frac{2\Lambda\bar{w}}{M+\Lambda}E_p(1-z)\chi_5+\frac{2M\bar{w}}{M+\Lambda}E_\Lambda(1-z)\chi_6\,, \\
4k^2tF_5 &= M\Lambda\chi_1+(E_pE_\Lambda+k^2)\chi_2+\frac{M}{\Lambda}\left(\frac{4E_pE_\Lambda}{1+z}+E_\Lambda{}^2+k^2\right)\chi_3+\left(\frac{4t}{1-z}-E_pE_\Lambda-k^2\right)\chi_4 \\
&\quad +\frac{2M\bar{w}}{M+\Lambda}[2E_p+E_\Lambda(1+z)]\chi_5-\frac{2\bar{w}E_\Lambda}{\Lambda(M+\Lambda)}[2t-(E_pE_\Lambda+k^2)(1-z)]\chi_6\,, \\
4k^2tF_6 &= \frac{4(\Lambda^2-M^2)}{\Lambda(1+z)}\chi_3+\frac{4t}{M+\Lambda}\chi_5+\frac{4Mt}{\Lambda(M+\Lambda)}\chi_6\,.
\end{aligned} \tag{7}$$

The amplitudes $\chi$ are readily projected into partial waves. We divide them into even and odd parts.

$$\begin{aligned}
\chi_i &= \Sigma_j(\chi_i{}^j)^+[d_{\lambda,\lambda'}{}^j(\theta)+(-1)^jd_{\lambda,\lambda'}{}^j(\theta)] \\
&\quad +(\chi_i{}^j)^-[d_{\lambda,\lambda'}{}^j(\theta)-(-1)^jd_{\lambda,\lambda'}{}^j(\theta)]\,,
\end{aligned} \tag{8}$$

where $(\chi_i{}^j)^\pm$ correspond to partial-wave helicity amplitudes with $j$ even and odd respectively, $\lambda$ and $\lambda'$ are the differences between the helicities of the proton and the antilambda for incoming and outgoing states. Equation (8) can be alternatively written as

$$\begin{aligned}
\chi_i &= \Sigma_j(\chi_i{}^j)^+(-1)^j[d_{\lambda,\lambda'}{}^j(\theta)+(-1)^\lambda d_{\lambda,-\lambda'}{}^j(\pi-\theta)] \\
&\quad +(\chi_i{}^j)^-(-1)^j[-d_{\lambda,\lambda'}{}^j(\theta)+(-1)^\lambda d_{\lambda,-\lambda'}{}^j(\pi-\theta)]\,,
\end{aligned} \tag{9}$$

where we have used the identity

$$(-1)^j d_{\lambda,\lambda'}{}^j(\theta)=(-1)^\lambda d_{\lambda,-\lambda'}{}^j(\pi-\theta)\,.$$

At this stage we would like to say something about the process associated with the exchange of a $K^*$ Regge pole. The fact that $K^*$ has odd signature and odd parity, limits the $N\bar{\Lambda}$ system to states with odd total angular momentum and even orbital angular momentum. Correspondingly, for a fixed value $j$ ($j$ is odd in this case), there can be only three different scattering amplitudes,

$$\begin{aligned}
&l=j+1 \text{ (triplet)} \leftrightarrow l=j+1 \text{ (triplet)}\,,\\
&l=j+1 \text{ (triplet)} \leftrightarrow l=j-1 \text{ (triplet)}\,,\\
&l=j-1 \text{ (triplet)} \leftrightarrow l=j-1 \text{ (triplet)}\,,
\end{aligned} \tag{10}$$

while the other three amplitudes

$$\begin{aligned}
&l=j \text{ (triplet)} \leftrightarrow l=j \text{ (triplet)}\,,\\
&l=j \text{ (triplet)} \leftrightarrow l=j \text{ (singlet)}\,,\\
&l=j \text{ (singlet)} \leftrightarrow l=j \text{ (singlet)}\,,
\end{aligned} \tag{11}$$

are equal to zero. In the language of partial-wave helicity amplitudes, (11) implies

$$\begin{aligned}
(\chi_1{}^j)^-&=(\chi_2{}^j)^-\,,\\
(\chi_3{}^j)^-&=(\chi_4{}^j)^-\,,\\
(\chi_5{}^j)^-&=(\chi_6{}^j)^-\,.
\end{aligned} \tag{12}$$

Returning now to Eq. (9) we note that since the $\chi$'s have the same analytic properties as the $F$'s—which, by assumption, satisfy the principle of maximal analyticity, i.e., all singularities are Regge poles[11]—the functions $\chi$ can be evaluated at the $K^*$ Regge pole. Combining with (12) we obtain:

$$\begin{aligned}
\chi_1=\chi_2&=\frac{-\pi}{\sin\pi\alpha}\beta_1(t)[P_\alpha(-z)-P_\alpha(z)]+L\,,\\
\chi_3&=\frac{-\pi}{\sin\pi\alpha}\beta_2(t)\left\{[P_\alpha(-z)-P_\alpha(z)]-\frac{1-z}{\alpha(\alpha+1)}[P_\alpha'(-z)+P_\alpha'(z)]\right\}+L\,,\\
\chi_4&=\frac{-\pi}{\sin\pi\alpha}\beta_2(t)\left\{-[P_\alpha(-z)-P_\alpha(z)]-\frac{1+z}{\alpha(\alpha+1)}[P_\alpha'(-z)+P_\alpha'(z)]\right\}+L\,,\\
\chi_5=-\chi_6&=\frac{-\pi}{\sin\pi\alpha}\beta_3(t)\frac{1}{\sqrt{\alpha(\alpha+1)}}[P_\alpha'(-z)+P_\alpha'(z)]+L\,,
\end{aligned} \tag{13}$$

where $\beta_1$, $\beta_2$, and $\beta_3$ are the residue functions of $(\chi_1{}^j)^-$, $(\chi_3{}^j)^-$, and $(\chi_5{}^j)^-$, respectively, at the $K^*$ Regge pole, and $L$ denotes the line integration from $-\frac{1}{2}-i\infty$ to $-\frac{1}{2}+i\infty$ which is negligible at high energy.

Had we considered a $K$ Regge pole, because of the even signature and odd parity, amplitudes (10) would vanish and (11) would be nonzero. Instead of (13), we would obtain

$$\begin{aligned}
\chi_1=-\chi_2&=\frac{-\pi}{\sin\pi\alpha}\beta_1(t)[P_\alpha(-z)+P_\alpha(z)]\,,\\
\chi_3&=\frac{-\pi}{\sin\pi\alpha}\beta_2(t)\left\{[P_\alpha(-z)+P_\alpha(z)]-\frac{1-z}{\alpha(\alpha+1)}[P_\alpha'(-z)-P_\alpha'(z)]\right\}\,,\\
\chi_4&=\frac{-\pi}{\sin\pi\alpha}\beta_2(t)\left\{[P_\alpha(-z)+P_\alpha(z)]+\frac{1+z}{\alpha(\alpha+1)}[P_\alpha'(-z)-P_\alpha'(z)]\right\}\,,\\
\chi_5=\chi_6&=\frac{-\pi}{\sin\pi\alpha}\beta_3(t)\frac{1}{\sqrt{\alpha(\alpha+1)}}[P_\alpha'(-z)-P_\alpha'(z)]\,,
\end{aligned} \tag{14}$$

where $\alpha$ and the $\beta$'s are now the Regge trajectory and residue functions of $K$.

By means of Eqs. (13) and (7) we can now obtain the scalar invariants $F$'s in terms of the $K^*$ Regge-pole residues and its trajectory. Combining with Eqs. (3) we obtain $\phi$'s in terms of the $K^*$ Regge pole.

[11] See, for instance, "Lectures on Strong Interaction Theory" given by Professor G. F. Chew at the Middle East Technical University and also at the University of Cambridge (unpublished).

$$
\begin{aligned}
4\pi W\phi_1&=\frac{4M^2u}{(t+u)(s+u)}A_1+A_2-\frac{st}{(t+u)(s+u)}A_3+\frac{4stu}{(t+u)(s+u)(s-u)}A_4,\\
4\pi W\phi_2&=\frac{-st}{(t+u)(s+u)}A_1+\frac{4M^2u}{(t+u)(s+u)}A_3+\frac{4stu}{(t+u)(s+u)(s-u)}A_4,\\
4\pi W\phi_3&=\frac{4M^2u}{(t+u)(s+u)}A_1-A_2-\frac{st}{(t+u)(s+u)}A_3+\frac{4stu}{(t+u)(s+u)(s-u)}A_4,\\
4\pi W\phi_4&=\frac{st}{(t+u)(s+u)}A_1-\frac{4M^2u}{(t+u)(s+u)}A_3-\frac{4stu}{(t+u)(s+u)(s-u)}A_4,\\
\frac{2M}{E\sin\theta}4\pi W\phi_5&=\frac{-4M^2}{s+u}A_1-\frac{4M^2}{s+u}A_3+\frac{2(4uM^2-st)}{(s+u)(s-u)}A_4,\\
\frac{2M}{E\sin\theta}4\pi W\phi_6&=\frac{4M^2}{s+u}A_1+\frac{4M^2}{s+u}A_3-\frac{2(4uM^2-st)}{(s+u)(s-u)}A_4,
\end{aligned}
\tag{15}
$$

where we have used a simple notation

$$A_1=\frac{-\pi}{\sin\pi\alpha}\beta_1(t)[P_\alpha(-z)-P_\alpha(z)],$$

$$A_2=\frac{-\pi}{\sin\pi\alpha}\beta_2(t)\left[\frac{-1}{\alpha(\alpha+1)}\right][P_\alpha'(-z)+P_\alpha'(z)],$$

$$A_3=\frac{-\pi}{\sin\pi\alpha}\beta_2(t)\{[P_\alpha(-z)-P_\alpha(z)]+\frac{z}{\alpha(\alpha+1)}[P_\alpha'(-z)+P_\alpha'(z)]\},$$

$$A_4=\frac{-\pi}{\sin\pi\alpha}\beta_3(t)\frac{z}{\sqrt{\alpha(\alpha+1)}}[P_\alpha'(-z)+P_\alpha'(z)].$$

In obtaining (15) we have neglected the mass difference between the proton and the lambda, which is possible because we are interested in the $s$ channel at high-energy and low-momentum transfer. It should be noticed that on the contrary this approximation is invalid if we are interested in the $t$ channel at large $s$ and small $t$.

With a lack of knowledge of the properties of the $K^*$ Regge pole, it is impossible to predict an explicit result, but the following can be noted.

(i) Since all $A$'s have the same phase, the polarization function $Q_{n_2}(\theta)$ vanishes. This in fact is a well-known result[12] for a single Regge-pole exchange. However, if we include a $K$ Regge pole also, the function $Q_{n_2}(\theta)$ will not vanish but will be small and of the order of $(s/2M^2)^{\alpha_K-\alpha_{K^*}}$ at high energy.

(ii) From (15) $\phi_2=-\phi_4$, $\phi_5=-\phi_6$ and $\phi_1$ differs from $\phi_3$ by the term $A_2$, which at high energy is one order of $(s/2M^2)$ smaller than the others. Using (4) it can be easily concluded that at high energy all the functions $R_{n_1\bar{n}_1}(\theta)$, $R_{n_3\bar{n}_3}(\theta)$, and $R_{n_1\bar{n}_3}(\theta)$ are of the order of $(s/2M^2)^{-1}$ which is small.

(iii) The differential cross section is proportional to

$$\frac{1}{s}\left(\frac{s}{2M^2}\right)^{2\alpha_{K^*}(t)},$$

where $\alpha_{K^*}(t)$ is the $K^*$ Regge trajectory, and might be approximated by $\alpha_{K^*}(t)=\alpha_{K^*}(0)+t\alpha_{K^*}'(0)$. The differential cross section decreases exponentially near the forward scattering. The width shrinks as $\ln s$ and the total cross section diminishes.

(iv) For forward scattering $\theta=0$, $\phi_4=0$ because of the conservation of angular momentum, and hence $\beta_2(\theta=0)=0$. If we assume that all the $\beta$'s are slowly varying functions of $t$, we may conclude $\beta_2=0$ at least for small values of $t$. Furthermore, if we assume that the residue functions can be factorized into spin-flip and spin-nonflip parts,[13] $\beta_1$, $\beta_2$, and $\beta_3$ are related by $\beta_3^2\sim\beta_1\beta_2$ and hence $\beta_3$ is also small. By (4) we then obtain that $R_{n_2\bar{n}_2}(\theta)$ is small.

## V. DISCUSSION

In the last two sections, results have been obtained on the differential cross section, polarization functions and angular correlation functions for the production $p+\bar{p}\to\Lambda+\bar{\Lambda}$ on the assumption that the process is dominated by the exchange of an elementary $K^*$ or a Regge-pole $K^*$. The differential cross sections are found in both cases to be strongly peaked forward, and to a good accuracy, they are experimentally indistinguishable. The angular correlation functions $R_{n_1\bar{n}_1}(\theta)$ $R_{n_1\bar{n}_3}(\theta)$ and $R_{n_3\bar{n}_3}(\theta)$ are nonvanishing and increase with the scattering angle in the case of an elementary $K^*$, while

[12] I. J. Muzinich, Phys. Rev. Letters 9, 475 (1962).

[13] W. G. Wagner, Phys. Rev. Letters 10, 202 (1963).

they are of the order of $(s/2M^2)^{-1}$, i.e., small in the case of a Regge-pole $K^*$. Since our results for the Regge-pole case are only valid at high-energy and low-momentum transfer, it is necessary to compare them with those obtained in the case of an elementary $K^*$ in the same region. In Fig. 5 we see that the functions $R_{n_1\bar{n}_1}(\theta)$, $R_{n_1\bar{n}_3}(\theta)$ have the same magnitude in the two cases, but $R_{n_3\bar{n}_3}(\theta)$ may be different. The total cross section for the case of a Regge-pole $K^*$ decreases as $\ln s$ and for the case of an elementary $K^*$ increases slowly.

In conclusion, with the present experimental data at 3–4 BeV/$c$ it seems impossible to distinguish between an elementary $K^*$ and a Regge-pole $K^*$, because in both cases, the predicted differential cross sections are strongly peaked forward and the angular correlation functions have the same magnitude for small values of $t$. However, if an additional experiment at a much higher energy (say around 7 or 8 BeV/$c$) is performed, by measuring the angular correlation function $R_{n_3\bar{n}_3}(\theta)$ and the total cross section, it may be possible to distinguish an elementary $K^*$ from a Regge-pole $K^*$.

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## APPENDIX

To obtain the relations between the helicity amplitudes and the scalar invariants, the dirac spinors of particle and antiparticle $u(p)$ and $v(p)$ are expressed in the c.m. system. When moving in the $(\sin\theta,0,\cos\theta)$ direction, we write

$$u(p)=\frac{1}{[2M(E+M)]^{1/2}}\begin{pmatrix}E+M\\ \sigma\cdot\mathbf{p}\end{pmatrix}|\chi\rangle$$

$$=\frac{1}{[2M(E+M)]^{1/2}}\begin{pmatrix}E+M\\ 2\lambda p\end{pmatrix}e^{-i\frac{1}{2}\theta\sigma_y}\begin{pmatrix}1\\0\end{pmatrix}\quad\text{or}\quad\begin{pmatrix}0\\1\end{pmatrix}$$

$$\text{for}\quad\lambda=\pm\tfrac{1}{2},\qquad\text{(A1)}$$

and

$$v(p)=\frac{1}{[2M(E+M)]^{1/2}}\begin{pmatrix}\sigma\cdot\mathbf{p}\\ E+M\end{pmatrix}|\chi\rangle$$

$$=\frac{1}{[2M(E+M)]^{1/2}}\begin{pmatrix}-2\lambda p\\ E+M\end{pmatrix}e^{-i\frac{1}{2}\theta\sigma_y}\begin{pmatrix}0\\1\end{pmatrix}\quad\text{or}\quad\begin{pmatrix}1\\0\end{pmatrix}$$

$$\text{for}\quad\lambda=\pm\tfrac{1}{2},\qquad\text{(A2)}$$

where $M$, $E$, and $p$ are the mass, energy, and momentum of the particle or antiparticle in the c.m. system; $u(p)$ and $v(p)$ satisfy the Dirac's equations $(i\gamma p+M)u(p)=0$ and $(-i\gamma\cdot p+M)v(p)=0$. We have taken the azimuth angle $\phi$ to be zero without losing any generality. Using (A1) and (A2) appropriately in the $T$ matrix, we obtain Eqs. (3) and (6).

To obtain Eqs. (4), we must also express the state of $\Lambda$ and $\bar{\Lambda}$ with spin polarized along $n_1$ or $n_2$ (see Fig. 3) in terms of helicity states. First let us consider the $\Lambda$ particle with spin polarized along $n_2$:

$$\langle\text{along } n_2|=\langle\text{along }\mathbf{q}_1|e^{-i\frac{1}{4}\pi(\sigma_x\cos\theta-\sigma_z\sin\theta)}$$

$$=(1,0)e^{i\frac{1}{2}\theta\sigma_y}e^{-i\frac{1}{4}\pi(\sigma_x\cos\theta-\sigma_z\sin\theta)}$$

$$=\frac{1}{\sqrt{2}}\{\langle\text{along }\mathbf{q}_1|-i\langle\text{along}-\mathbf{q}_1|\};$$

similarly, $\langle\text{along } n_1|=\langle\text{along }\mathbf{q}_1|e^{i\frac{1}{4}\pi\sigma_y}$

$$=\frac{1}{\sqrt{2}}\{\langle\text{along }\mathbf{q}_1|+\langle\text{along}-\mathbf{q}_1|\};$$

also, for the polarized $\bar{\Lambda}$ particle (which is moving in the $-\mathbf{q}_1$ direction) we get:

$$|\text{along } n_2\rangle=e^{-i\frac{1}{4}\pi(\sigma_x\cos\theta-\sigma_z\sin\theta)}|\text{along}-\mathbf{q}_1\rangle$$

$$=e^{-i\frac{1}{4}\pi(\sigma_x\cos\theta-\sigma_z\sin\theta)}e^{-i\frac{1}{2}(\theta+\pi)\sigma_y}\begin{pmatrix}0\\1\end{pmatrix}$$

$$=\frac{1}{\sqrt{2}}\{|\text{along}-\mathbf{q}_1\rangle+i|\text{along}-\mathbf{q}_1\rangle\},$$

$$|\text{along } n_1\rangle=e^{i\frac{1}{4}\pi\sigma_y}|\text{along}-\mathbf{q}_1\rangle$$

$$=\frac{1}{\sqrt{2}}\{|\text{along}-\mathbf{q}_1\rangle+|\text{along }\mathbf{q}_1\rangle\}.$$

Writing the results in a compact form, for the outgoing polarized $\Lambda$,

$\langle$having spin along and opposite to $n_1$, $n_2$, $n_3$, respectively,$|$

$$=\frac{1}{\sqrt{2}}\{\langle+|\pm\langle-|\},\quad\frac{1}{\sqrt{2}}\{\langle+|\mp i\langle-|\},\quad\langle\pm|\qquad\text{(A3)}$$

and for outgoing polarized $\bar{\Lambda}$,

$|$Having spin along and opposite to $n_1$, $n_2$, $n_3$, respectively,$\rangle$

$$=\frac{1}{\sqrt{2}}\{|+\rangle\pm|-\rangle\},\quad\frac{1}{\sqrt{2}}\{|+\rangle\pm i|-\rangle\},\quad|\mp\rangle,\qquad\text{(A4)}$$

where $|+\rangle$ and $|-\rangle$ are the helicity states of $\Lambda$ or $\bar{\Lambda}$

with helicities $\lambda=\pm\frac{1}{2}$. Expressions (A3) and (A4) are correct up to a phase factor which is not important. With the help of (A3) and (A4), it is now easy to obtain all the equations of (4). We shall show below the calculation of $R_{n_1\bar{n}_1}(\theta)$ only.

Let us define $A$, $B$, $C$, $D$ to be the four helicity amplitudes with well-defined incoming $p\bar{p}$ helicities and with $++$, $+-$, $-+$, $--$ helicities for the outgoing $\Lambda\bar{\Lambda}$, where $A$, $B$, $C$, $D$ can be any column of the matrix in Table I. Hence, the helicity amplitudes for polarized $\Lambda\bar{\Lambda}$ will be as follows

$\Lambda$ along $n_1$, $\bar{\Lambda}$ along $n_1$: $\theta_1=\frac{1}{2}[(A+C)+(B+D)]$,

$\Lambda$ along $n_1$, $\bar{\Lambda}$ along $-n_1$: $\theta_2=\frac{1}{2}[(A+C)-(B+D)]$,

$\Lambda$ along $-n_1$, $\bar{\Lambda}$ along $n_1$: $\theta_3=\frac{1}{2}[(A-C)+(B-D)]$,

$\Lambda$ along $-n_1$, $\bar{\Lambda}$ along $-n_1$: $\theta_4=\frac{1}{2}[(A-C)-(B-D)]$.

The probability of finding $\Lambda$ polarized along $n_1$ and simultaneously $\bar{\Lambda}$ polarized along $-n_1$ is now given by

$$\frac{|\theta_2|^2+|\theta_3|^2-|\theta_1|^2-|\theta_4|^2}{|\theta_1|^2+|\theta_2|^2+|\theta_3|^2+|\theta_4|^2},$$

which is equal to

$$\frac{-2\,\mathrm{Re}[AD^*+BC^*]}{|A|^2+|B|^2+|C|^2+|D|^2}.$$

If we assume that the beam and the target are unpolarized, we can sum all these 4 columns of the matrix in Table I and to obtain

$$R_{n_1\bar{n}_1}(\theta)=\frac{2\,\mathrm{Re}[\phi_6\phi_6^*-\phi_3\phi_4^*-\phi_1\phi_2^*-\phi_5\phi_5^*]}{|\phi_1|^2+|\phi_2|^2+|\phi_3|^2+|\phi_4|^2+2|\phi_5|^2+2|\phi_6|^2},$$

i.e., the one given in (4). Similarly we can derive the remaining expressions in Eqs. (4).

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# Inelastic $\pi^+\pi^-$, $K^0\bar{K}^0$, and $K^+K^-$ Collisions in Unitary Symmetry*

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The $G$ conjugation invariance in the isospin representation and its analogs in the $u$-spin and $v$-spin representations of unitary symmetry are applied to the inelastic channels of reactions initiated in the $\pi^+\pi^-$, $K^0\bar{K}^0$, and $K^+K^-$ systems. Relations among differential cross sections for various particle-antiparticle pairs are obtained with and without $R$ invariance.

## INTRODUCTION

THE object of this paper is to obtain relations among the cross sections for various particle-antiparticle pairs in the inelastic channels of reactions initiated in collisions of $\pi^+$ and $\pi^-$, $K^0$ and $\bar{K}^0$, and $K^+$ and $K^-$ within the scheme of unitary symmetry.[1,2] For reactions initiated in the $\pi^+\pi^-$ system the eigenstates of pairs of pseudoscalar mesons, vector mesons, baryon and antibaryon, and baryon isobar and antibaryon isobar are constructed under the conservation of isospin and $G$ conjugation invariance.[3] Relations among the cross sections for the possible pairs are then immediately obtained.

For reactions initiated in the $K^0\bar{K}^0$ system, the isospin representation cannot be used to carry out the same program since it is not an eigenstate of $G$ conjugation. In order to do so it is found necessary to go to the $u$-spin representation[4] of unitary symmetry and invoke the analog of $G$ conjugation (henceforth called $U$ conjugation) of which the $K^0\bar{K}^0$ system is an eigenstate.

For reactions initiated in the $K^+K^-$ system, the previous representations cannot be used to carry out the program since it is not an eigenstate of the $G$ nor of the $U$ conjugation. It is then found necessary to go to the $v$-spin representation[4] and invoke the analog of the $G$ and $U$ conjugations (henceforth called $V$ conjugation) of which the $K^+K^-$ system is an eigenstate. These results can then be correlated to each other since the isospin, the $u$-spin, and the $v$-spin representations are canonically related to one another under unitary symmetry.

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